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Certainly! Here is a revised, well-formatted version of the provided Multifractal Asset Pricing Model paper in markdown. The formatting includes clear section headers, bullet points, summary tables, code-style analytical formulas, and improved readability suitable for publication or presentation. Citations reference the original source file for every section.[[1]](#fn1)

**Multifractal Asset Pricing Model: Unified Framework for Derivative Valuation and Market Efficiency**

**Running Head:** Multifractal Asset Pricing Model  
**Keywords:** Stable distributions, fractional Brownian motion, multifractal processes, derivative pricing, Kelly criterion  
**JEL Classifications:** G12, G13, C58, C61  
**Authors:** [To be completed]  
**Source:**[[1]](#fn1)

**Abstract**

This paper introduces the Multifractal Asset Pricing Model (MAPM), a unified framework integrating Pareto-Lévy stable distributions (α), fractional Brownian motion (H), and multifractal theory (λ). MAPM derives universal and derivative-specific pricing formulas using only three parameters and empirically validates parameter constancy and variation on NASDAQ 100 derivatives. Twelve fundamental theorems establish rigorous mathematical integration, fundamentally advancing market modeling and efficiency.[[1]](#fn1)

**Introduction: Unifying Three Traditions**

**The Crisis in Traditional Derivative Modeling**

* Existing models (Black-Scholes, Heston, local volatility, jump-diffusion) are fragmented and parameter-heavy.
* No unified theoretical foundation; model proliferation to solve isolated empirical failures.[[1]](#fn1)

**The MAPM Revolution**

MAPM integrates three established mathematical traditions:

* **Pareto-Lévy Stable Distribution Theory** (α): Controls tail thickness; convolution stability; universal across derivatives.
* **Fractional Brownian Motion** (H): Determines long-range dependence; path persistence; varies by derivative sampling.
* **Multifractal Theory** (λ): Measures volatility clustering; regime shifts; varies by path-dependence.[[1]](#fn1)

**Mathematical Foundations**

**Pareto-Lévy Stability Index**

* **Convolution Theorem:** α must be identical across all derivatives for no-arbitrage pricing.
* **Empirical Bounds:** 1.5 <= α <= 1.8 in financial data; NASDAQ 100 has α = 1.8 ± 0.034 for all derivatives.[[1]](#fn1)

**Table: Three-Framework Parameter Integration**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Parameter | Theory Source | Symbol | Range | NASDAQ 100 | Role | Derivative Consistency |
| Stability Index | Pareto-Lévy | α | [[1]](#fn1) | 1.8 ± 0.034 | Heavy tails, stability | Must be identical |
| Hurst Exponent | Frac. Brownian Motion | H | (0,1) | 0.55 ± 0.023 | Long-range dependence | Varies by sampling |
| Intermittency | Multifractal Theory | λ | [0,∞) | 0.32 ± 0.124 | Vol clustering | Varies by path-depend. | [[1]](#fn1) |

**Fractional Brownian Motion and Derivative-Specific Sampling**

* **H values:**
  + H = 0.5: No memory
  + H > 0.5: Persistence
  + H < 0.5: Mean-reverting

**Derivative Effects**

* **European:** H ≈ H\_underlying
* **Asian:** H < H\_underlying (averaging reduces persistence)
* **Barrier/Lookback:** H > H\_underlying (amplifies extremes).[[1]](#fn1)

**Multifractal Theory and Path-Dependence**

* **λ values:**
  + λ = 0: Monofractal
  + λ > 0: Multifractal clustering

**Derivative Variations**

* **European:** λ ≈ λ\_underlying
* **Asian:** λ < λ\_underlying (averaging smooths volatility)
* **Barrier/Digital:** λ > λ\_underlying (amplifies clustering extremes).[[1]](#fn1)

**Lambda Regimes and MaxEnt Analysis**

**Table: Lambda Regime Classification**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Regime | λ Range | Frequency | Dominant Framework | Market State | Parameter Variation |
| I | 0 ≤ λ ≤ 0.2 | 16.2% | Fractional Brownian | Efficient trends | Minimal |
| II | 0.2 < λ ≤ 0.6 | 68.4% | Balanced Integration | Normal clustering | Significant |
| III | λ > 0.6 | 15.4% | Multifractal Theory | Crisis, extremes | Amplified | [[1]](#fn1) |

**Characteristic Functions and Density Recovery**

* **Closed-form PDFs/CDFs**: Unavailable in most α ranges (Zolotarev 1986).
* **Characteristic Function Approach:** Density recovered via FFT using derivative-specific parameters: α universal, H and λ custom.[[1]](#fn1)

**Fundamental Theorems**

**Table: Fundamental Theorems with Parameter Scope**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| # | Scope | Key Result | Empirical Test | Validated |
| 1 | All parameters | S\_q(τ)=C\_q τ^{qH-λq(q-1)/2} | Structure function analysis | ✓ |
| 2 | α only | α\_underlying = α\_derivative | Cross-derivative F-test | ✓ |
| 3 | H & λ variable | H,λ = f(sampling, path-dependence) | Derivative-specific t-test | ✓ |
| 4 | All params | E[r\_t]→0 under optimization (Kelly) | Kelly Beta Test | ✓ |
| 5 | H & λ | Predictable functional relationships | R^2 > 0.85 regression | ✓ | [[1]](#fn1) |

**Contingent Claim Partitioning**

* **Universal α:** Convolution theorem requires identical α.
* **H & λ:** Vary by derivative sampling, path-dependence.
* Each derivative prices as a claim on a segment of multifractal returns.[[1]](#fn1)

**Table: MAPM vs. Traditional Model Comparison**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Feature | Black-Scholes | Heston | Local Vol | Jump-Diff | MAPM |
| # Parameters | 1 (σ) | 5 | 100+ | 6-8 | 3 (α, H, λ) |
| α Consistency | No | No | No | No | Yes (universal) |
| H Variation | No | No | No | No | Yes (custom) |
| λ Adaptation | No | No | No | No | Yes (custom) |
| Internal Consistency | No | No | No | No | Yes |
| Regime Recognition | No | No | No | Limited | Yes (3 regimes) |
| Parameter Stability | Poor | Poor | Very Poor | Poor | Excellent |
| Crisis Performance | Fails | Fails | Fails | Moderate | Robust | [[1]](#fn1) |

**Example: Derivative-Specific Parameter Relationships**

**European Call**

* α\_European = α\_underlying
* H\_European ≈ H\_underlying
* λ\_European ≈ λ\_underlying

**Asian Option**

* α\_Asian = α\_underlying
* H\_Asian < H\_underlying
* λ\_Asian < λ\_underlying

**Barrier Option**

* α\_Barrier = α\_underlying
* H\_Barrier > H\_underlying
* λ\_Barrier > λ\_underlying[[1]](#fn1)

**Empirical Results: NASDAQ 100 Validation**

**Sample, Estimation Strategy**

* NASDAQ 100, 1998–2025, all major derivatives.
* Universal α estimated (maximum likelihood); H, λ calibrated per derivative via structure function and multifractal analysis.[[1]](#fn1)

**Table: NASDAQ 100 Parameter Validation**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Test Category | Specific Test | Result | Significance | Interpretation |
| α Consistency | Cross-derivative F-test | F=1.23, p=0.31 | No rejection | Confirms theory |
| H Variation | Asian < Underlying | t=-3.47, p<0.001 | Significant | Less persistence |
| H Variation | Barrier > Underlying | t=4.23, p<0.001 | Significant | Amplifies trends |
| λ Variation | Asian < Underlying | t=-2.89, p=0.004 | Significant | Less clustering |
| λ Variation | Barrier > Underlying | t=3.15, p=0.002 | Significant | Amplifies clusters |
| Relationships | Parameter R^2 | All>0.85 | Highly significant | Predictable forms | [[1]](#fn1) |

**Derivative-Specific Parameters**

|  |  |  |  |
| --- | --- | --- | --- |
| Derivative Type | H Value | λ Value | Relationship |
| Underlying | 0.547 ± 0.023 | 0.324 ± 0.124 | Baseline |
| European | 0.545 ± 0.025 | 0.321 ± 0.118 | ≈ Underlying |
| Asian | 0.493 ± 0.031 | 0.267 ± 0.098 | Reduced by averaging |
| Barrier | 0.584 ± 0.019 | 0.389 ± 0.142 | Amplified sensitivity |
| Digital | 0.549 ± 0.027 | 0.328 ± 0.126 | Minimal modification | [[1]](#fn1) |

**Pricing Accuracy**

|  |  |  |  |
| --- | --- | --- | --- |
| Model | Parameter Structure | RMSE | Improvement vs MAPM |
| MAPM | α universal, H/λ variable | 0.732 | Benchmark |
| MAPM-Fixed | α, H, λ all constant | 0.891 | 18% worse |
| Black-Scholes | Gaussian assumptions | 1.224 | 40% worse |
| Heston | Stochastic volatility | 1.087 | 33% worse | [[1]](#fn1) |

**Conclusion & Impact**

* **Universal α:** Empirically validated, required by convolution stability.
* **Variable H & λ:** Enhance empirical realism and pricing accuracy without violating theory.
* **MAPM eliminates fragmented modeling; enables robust risk management and innovative derivatives.**
* **Framework supports future research, multi-asset applications, and regulatory innovation**.[[1]](#fn1)

**References**

* Zolotarev 1986, Nolan 2020, Samorodnitsky & Taqqu 1994 (Stable distributions)[[1]](#fn1)
* Mandelbrot & Van Ness 1968, Hurst 1951 (Fractional Brownian motion)[[1]](#fn1)
* Mandelbrot 1997, Muzy et al. 2001, Bacry et al. 2001 (Multifractal theory)[[1]](#fn1)
* Black & Scholes 1973, Heston 1993, Merton 1976 (Traditional pricing)[[1]](#fn1)
* Kelly 1956, Thorp 2006 (Kelly criterion, optimal trading)[[1]](#fn1)

**Planned Figures**

* **Figure 1:** Schematic: α universal, H/λ variable across derivatives.
* **Figure 2:** α clustering, cross-derivative F-test.
* **Figure 3:** Box-plots: H variation by derivative type.
* **Figure 4:** Violin plots: λ variation and path-dependence.
* **Figure 5:** Regression: functional parameter relationships.
* **Figure 6:** Pricing accuracy improvement – RMSE comparison.
* **Figure 7:** Rolling window: 27-year parameter stability.
* **Figure 8:** Regime analysis: λ regimes and parameter effects.[[1]](#fn1)

**Appendices Outline**

* **Mathematical derivations:** α consistency, H & λ flexibility proofs.
* **Empirical parameter estimation:** Maximum likelihood, structure function, detrended fluctuation.
* **Validation framework:** F-tests, t-tests, regression.
* **Numerical implementation:** Zolotarev, FFT, Greeks.
* **Risk management:** Portfolio/derivative-level, regime-specific hedges.
* **Cross-asset validation:** Extensions to global assets.
* **Dynamic parameter models:** Regime-switching, forecasting.[[1]](#fn1)

This version preserves the content and technical rigor while presenting the arguments, evidence, formulas, and empirical results in an organized, accessible, and visually appealing markdown format suitable for review and further editing.[[1]](#fn1)

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1. [JFIN-Article-Latest-version-0.3-v0.1.md](http://JFIN-Article-Latest-version-0.3-v0.1.md)